Logarithm Reference Page

Logarithm Definitions

\[ \log_b(M) = N \text{ means } b^N = M \]

Since we usually use logarithms to base 10 (common logarithms) or base \( e \) (natural logarithms), the number \( b \) in the definition is usually \( e \) or 10:

\[ \log_e(M) = \ln(M) = N \text{ means } e^N = M \]
\[ \log_{10}(M) = \log M = N \text{ means } 10^N = M \]

The logarithm is an extremely slow-growing function. For example, \( \log(100) = 2 \) since \( 10^2 = 100 \); for the common logarithm function to reach 3, we have to wait for an input of 1000; to reach 4, we have to wait until 10,000; and so on. In fact, the common logarithm tells us the number of digits needed to write out a number: if \( \log(x) = 6.485729 \), for example, we know \( x = 10^{6.485729} \), which is between 10⁶, 1 million, and 10⁷, 10 million, so a 7-digit number.

- **Properties based on the definition:**
  \[ \log_b(b^N) = N \]
  \[ b^{\log_b(M)} = M \]

- **Multiplication and Division INSIDE a logarithm:**
  \[ \log_b(P \times Q) = \log_b P + \log_b Q \]
  \[ \log_b\left(\frac{P}{Q}\right) = \log_b P - \log_b Q \]

- **Exponents inside a logarithm**
  \[ \log_b(P^n) = n \log_b(P) \]

- **Change of Base** (useful for calculating \( \log_3(17) \) on a calculator that only has \( \log_{10} \) and \( \ln \) buttons)
  \[ \log_b(M) = \frac{\log(M)}{\log(b)} = \frac{\ln(M)}{\ln(b)} \]

- **Using the Properties to Solve an Exponential Equation** To solve \( 5^x = 8 \), take logs of both sides:
  \[ \log(5^x) = \log 8, \]
  then use the exponent rule
  \[ x \times \log(5) = \log 8, \]
  and divide:
  \[ x = \frac{\log 8}{\log 5} \ (= \log_5(8) \ ) \]

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