This is a question with answer to an old AMATYC test problem.

**Question**  Find the value of the expression

\[ x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}} } \]

**Answer.** Since squaring \( \sqrt{z} \) gives \( z \), when we square \( x \) we just remove the outside square root sign. So

\[ x^2 = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}} } \]

That is, \( x^2 = 2 + x \), since the complicated expression with square root signs is \( x \). Thus we look for solutions to the quadratic equation

\[ x^2 - x - 2 = 0 . \]

This factors: we have

\[(x - 2)(x + 1) = 0 \]

so either \( x = 2 \) or \( x = -1 \) — but the second is impossible, because \( x \) is a square root; thus \( x = 2 \).

*Sam Needham ©2002*