1. Consider the following initial value problem.
\[ \frac{dy}{dt} = y^2 t \quad y(0) = 2 \]
Are the conditions for existence, uniqueness met? Explain. State the conditions. Find and sketch the general solution. Solve the initial value problem and indicate the solution curve on your sketch. State the domain of definition.

2. \[ \frac{dy}{dt} = 2y - y^3 \]
Sketch the solution curves, DE graph and phase line. Identify and classify all equilibrium solutions. Describe the long term behavior \( t \to \pm\infty \) in all regions.

3. \[ \dot{x} = x(y - 2) \]
\[ \dot{y} = (x - 2)y \]
Find all equilibrium points. Draw the horizontal and vertical tangent lines in the phase plane. For each of the nine regions defined by your nullclines, indicate with one big arrow the direction of the vector field. For each of the equilibrium points, find the linearized system. Classify the equilibrium points and state whether they are hyperbolic or not. State what you can infer about the local behavior of the nonlinear system near each equilibrium point. Sketch the local phase portraits in the neighborhood of the equilibrium points. Include arrows on your phase curves. Sketch the global phase portrait. Indicate the separatrix on your phase portrait. No equation is necessary. If you didn't find a separatrix, you made a mistake. Go back and find it. Describe the global behavior for the system.

4. \[ \ddot{y} + b\dot{y} + 9y = \sin \omega t \quad y(0) = 0, \quad \dot{y}(0) = 0 \]
For what values of \( b \) and \( \omega \) is the solution periodic? For what values are there frequency beats? Solve the system in the resonant case and sketch the solution.
If \( b = 2 \) describe the long term behavior of the system. YOU NEED NOT SOLVE.

5. \[ \dot{x} = y \]
\[ \dot{y} = -x + \alpha x^3 + \beta y \]
For what values of the parameters \( \alpha \) and \( \beta \) is the system Hamiltonian. Find the Hamiltonian
For the Hamiltonian case only show that \( \alpha = 0 \) is a bifurcation value. Sketch the phase portraits in the Hamiltonian case for the appropriate parameter values.