Practice Exam 2  Differential Equations  Prof. L. A. Month

1. Match the phase portraits on the next page with the linear systems given below. Do this by writing the letter of the appropriate system in the box under each phase portrait.

A. \[ \dot{x} = 3x - y \]
B. \[ \dot{x} = -2x - 2y \]
C. \[ \dot{x} = 5x + 4y \]
D. \[ \dot{x} = y, \quad \dot{y} = -3x + y \]
E. \[ \dot{x} = y, \quad \dot{y} = -x \]
F. \[ \dot{x} = x, \quad \dot{y} = y \]
G. \[ \dot{x} = 2x, \quad \dot{y} = 3y \]

2. Consider the following harmonic oscillators with periodic forcing. For each oscillator find and describe the "steady state" response. Your description should include discussions of resonance, frequency beats and phase. If the steady state response is periodic find its amplitude, phase and frequency, unless there are frequency beats. If the response has frequency beats indicate the frequency of the beats and draw a sketch labeling the appropriate frequencies. If the steady state response is not periodic but oscillatory find the frequency of the oscillations.

a. \[ y + 2\dot{y} + 9y = \sin 3t, \quad y(0) = 0, \quad \dot{y}(0) = 0 \]
b. \[ y + 9y = \sin 3t, \quad y(0) = 0, \quad \dot{y}(0) = 0 \]
c. \[ y + 9y = \cos 3.1t, \quad y(0) = 0, \quad \dot{y}(0) = 0 \]

3. Find the general solution to the planar linear system using eigenvalues and eigenvectors.

\[ \dot{Y} = AY \]
\[ A = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix} \]
\[ Y = \begin{pmatrix} x \\ y \end{pmatrix} \]

Sketch the phase portrait.
Label the initial condition and solution curve for \( x(0) = 4 \), \( y(0) = -3 \).
Sketch the \( x(t) \) and \( y(t) \) graphs for this particular solution. Plot \( x(t) \) and \( y(t) \) on the same graph.

4. Consider a one parameter family of linear systems depending on the parameter \( a \)

\[ \dot{Y} = \begin{pmatrix} 0 & a \\ -2 & 3 \end{pmatrix} Y \]
\[ Y = \begin{pmatrix} x \\ y \end{pmatrix} \]

a. The family determines a curve in the trace determinant plane. Sketch the curve in the trace determinant plane.
b. Identify the values of \( a \) where the type of system changes. These are the bifurcation values.
c. Briefly discuss the behavior in each of the regions of the trace-determinant plane that the curve traverses, including the bifurcation value(s). In your discussion you should classify the equilibrium point(s) and describe the long term behavior \( (t \to \pm \infty) \), as well as providing a rough sketch of the phase portraits with arrows. No specific calculations with particular values of \( a \) are necessary except for the bifurcation value(s). In a horizontal row present a series of phase portraits for increasing \( a \). Give one portrait per region and one portrait for each bifurcation value with the portraits arranged in order of increasing \( a \).