1. Define $f$ by $f(x) = \lfloor \log_2 x \rfloor$.
   a) What is the domain of $f$?
   b) Is $f$ 1-1?
   c) Is $f$ onto the integers?
   d) What is the range of $f$?
   e) Does $f$ have an inverse? If so, find $f^{-1}$.
   f) Draw the graph of $f$.

2. $f(x) = \log_2 \left( 1 + \frac{1}{x} \right)$
   a) Prove $f(x)$ is decreasing from the definition of decreasing functions.
   b) What is the domain of $f$?
   c) Find the range of $f$. Is $f$ onto $\mathbb{R} - \{0\}$?
   d) Find $O(f(x))$. (Best Big Oh)
   e) Prove $f(x)$ is 1-1.

3. Solve the recurrence relation
   $$a_k - 2 + 3a_{k-1} + 3a_k = 0 \quad k \geq 2$$
   $$a_0 = 0$$
   $$a_1 = 2$$

4. Let $m_n$ be the maximum number of comparisons for a merge sort algorithm with an input of $n$ elements.
   a) Write the recurrence relation for $m_n$.
   b) Prove by induction: $m_n \leq 2n \log_2 n \quad n \geq 1$
   c) Derive the order of the merge sort algorithm from this result.

5. Input an array of 8 elements: 5, 6, 4, 7, 2, 9, 3, 1.
   a) Sort by insertion sort. Make a table to show the results at each stage. Count the number of comparisons.
   b) Repeat for merge sort.
   c) Next search for the element 2 in your sorted list. As before, present a table and count comparisons. Analyze using sequential search and then repeat for binary search.
   d) List the total number of comparisons for the 4 combinations:
      Insertion sort, binary search.
      Insertion sort, sequential search.
      Merge sort, binary search.
      Merge sort, sequential search.
   e) Derive the order for each of the 4 combinations of algorithms. You may use the orders for the individual algorithms that were given in class or the text. (e.g., sequential search is $O(n)$.)

Please sign:

This exam is entirely my own work. I did not consult with anyone else for help with the exam.