Exam 4  Calculus 2  M193-1240  Prof. L. A. Month

(05%)  1. Find the second partial sum of the infinite series. Find the sum of the infinite series.

\[ \sum_{n=0}^{\infty} \frac{5}{2^n} - \frac{1}{3^n} \]

(05%)  2. Use the integral test to determine if the series converges or diverges. Be sure to check that the conditions of the integral test are satisfied.

\[ \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \]

(30%)  3. Use any method to determine if the series converges or diverges. Give reasons.

\[ \sum \frac{n^2 + 2n + 5}{n^4 - 3n^2 + 5} \]
\[ \sum \frac{(\ln n)^2}{n^{3/2}} \]
\[ \sum \frac{(2n)!}{n! \ n!} \]
\[ \sum \frac{n! \ n! \sqrt{n}}{(2n)!} \]
\[ \sum \frac{2^{n^2}}{n!} \]
\[ \sum \frac{3^n}{4n} \]

(10%)  4. Determine if the alternating series converges or diverges.

\[ \sum \frac{(-1)^{n+1}}{\sqrt{n}} \]
\[ \sum \frac{(-1)^{n+1}3\sqrt{n} + 1}{\sqrt{n} + 1} \]

(10%)  5. For each series determine if it converges absolutely or converges conditionally or diverges.

\[ \sum (-1)^n \frac{1 + n}{n^2} \]
\[ \sum \frac{(-100)^n}{n!} \]
6. Bound the magnitude of the error in using the sum of the first 4 terms to approximate the sum of the entire series. Is the estimate an underestimate or an overestimate? Explain.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{10^n} \]

7. Find the series radius and interval of convergence. For what values of \( x \) does the series converge absolutely? For what values of \( x \) if any does the series converge conditionally?

\[ \sum_{n=1}^{\infty} \frac{4^n x^{2n}}{n} \]
\[ \sum_{n=1}^{\infty} \frac{(-1)^n n! (x - 2)^n}{5^n} \]
\[ \sum_{n=1}^{\infty} \frac{3^n x^n}{n!} \]

8. Find the Taylor series of \( f(x) = \ln (2 + 5x) \) about \( x = 1 \) in closed form.

9. Sum the series. Leave your answer in exact form.

\[ 1 + 2 \left( \frac{\pi}{4} \right) + 3 \left( \frac{\pi}{4} \right)^2 + 4 \left( \frac{\pi}{4} \right)^3 + \cdots \]

10. Write the first four terms in the binomial series expansion of \( \sqrt{5 - 2x} \).