1. The force required to maintain a nonlinear spring stretched $x$ units beyond its natural length is given by $F(x) = 6x - x^3$.

A spring is stretched from $\sqrt{2}$ units beyond its natural length to $\sqrt{3}$ units beyond its natural length. Find the average value of the spring force required to maintain the spring. Leave your answer in exact form.

Bonus
Verify the mean value theorem of integrals by showing that the average spring force occurs for some $x$ between $\sqrt{2}$ and $\sqrt{3}$. You will need to use your calculator.

2. A stainless steel tank is formed by revolving the curve $y = x^2$ from $x = 0$ to $x = 4$ about the $y$-axis. The tank is filled with sea water to a depth of 3m from the bottom. Find the work done in pumping the sea water to the top of the tank. The density of sea water is $\frac{1000 \text{N}}{\text{m}^3}$. Leave your answer in exact form. Leave $\pi$ unchanged. Include the units.

3. Using the method of disks/washers or shells find the volume of the initial amount of sea water in the tank described in problem 2. Leave your answer in exact form. Leave $\pi$ unchanged. Include the units.

Bonus
Find the volume using the method you did not choose and verify it is the same as the volume you found above. Leave your answer in exact form. Leave $\pi$ unchanged. Include the units.

4. Use symmetry and the formula for arc length to set up the integral to find the circumference of the ellipse $x^2 + 2y^2 = 1$. Simplify the integrand as much as possible. Do not evaluate the integral.

5. A 100 ft cable weighs 250 lb and hangs vertically from the top of a tall building. Find the work done to lift the cable to the top of the building.

Bonus:
A leaky 20 lb bucket is lifted from the ground to a height of 100 ft by the cable described above. Initially the bucket contain 10 lb of olive oil but the olive oil leaks at a constant rate and finishes draining just as the bucket reaches the top. Find the work done.

6. Find the area of the region in the first quadrant enclosed by $y = x^2$, $x = y^2$, $x = 0$ and $x = 2$. Leave your answer in exact form. Simplify as much as possible.