Hyperbolic functions

\[ y = \sinh x = \frac{e^x - e^{-x}}{2} \]

Domain \((-\infty, \infty)\)
Range \((-\infty, \infty)\)
Odd \(f(-x) = -f(x)\)
Derivative \(\cosh x\)

\[ y = \cosh x = \frac{e^x + e^{-x}}{2} \]

Domain \((-\infty, \infty)\)
Range \([1, \infty)\)
Even \(f(-x) = f(x)\)
Derivative \(\sinh x\)

\[ y = \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \]

Domain \((-\infty, \infty)\)
Range \((-1, 1)\)
Odd \(f(-x) = -f(x)\)
Derivative \(\text{sech}^2 x\)

\[ y = \sinh^{-1} x = \ln (x + \sqrt{x^2 + 1}) \]

Domain \((-\infty, \infty)\)
Range \((-\infty, \infty)\)
Odd \(f(-x) = -f(x)\)
Derivative \(\frac{1}{\sqrt{1+x^2}}\)

\[ y = \cosh^{-1} x = \ln (x + \sqrt{x^2 - 1}) \]

Domain \([1, \infty)\)
Range \([0, \infty)\)
Even \(f(-x) = f(x)\)
Derivative \(\frac{1}{\sqrt{x^2-1}}\)

\[ y = \tanh^{-1} x = \frac{1}{2} \ln \left( \frac{x + 1}{1-x} \right) \]

Domain \((-1, 1)\)
Range \((-\infty, \infty)\)
Odd \(f(-x) = -f(x)\)
Derivative \(\frac{1}{1-x^2}\)
Hyperbolic functions

\[ y = \cosh x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} \]

Domain \((-\infty, 0) \cup (0, \infty)\)
Range \((-\infty, \infty)\)
Odd \(f(-x) = -f(x)\)
Even \(f(-x) = f(x)\)
Derivative \(-\csc x \coth x\)

\[ y = \sinh x = \frac{2}{e^x + e^{-x}} \]

\[ y = \tanh x = \frac{1}{e^x - e^{-x}} \]

\[ y = \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \]

Domain \((-\infty, 0) \cup (0, \infty)\)
Range \((-\infty, -1) \cup (1, \infty)\)
Odd \(f(-x) = -f(x)\)
Even \(f(-x) = f(x)\)
Derivative \(\csc^2 x\)

\[ y = \cosh^{-1} x = \sinh^{-1} \left(\frac{1}{x}\right) = \ln \left(\frac{1 + \sqrt{1 + x^2}}{|x|}\right) \]

Domain \((-\infty, 0) \cup (0, \infty)\)
Range \((-\infty, 0) \cup (0, \infty)\)
Odd \(f(-x) = -f(x)\)
Derivative \(-\frac{1}{x \sqrt{1 - x^2}}\)

\[ y = \tanh^{-1} x = \cosh^{-1} \left(\frac{1}{x}\right) = \ln \left(\frac{1 + \sqrt{1 - x^2}}{x}\right) \]

Domain \((0,1]\)
Range \([0, \infty)\)
Odd \(f(-x) = -f(x)\)
Derivative \(-\frac{1}{x \sqrt{1 - x^2}}\)

\[ y = \coth^{-1} x = \tanh^{-1} \left(\frac{1}{x}\right) = \frac{1}{2} \ln \left(\frac{x + 1}{x - 1}\right) \]

Domain \((-\infty, -1) \cup (1, \infty)\)
Range \((-\infty, 0) \cup (0, \infty)\)
Odd \(f(-x) = -f(x)\)
Even \(f(-x) = f(x)\)
Derivative \(\frac{1}{1 - x^2}\)